

QUATERNION NORMALIZATION IN SPACECRAFT ATTITUDE DETERMINATION

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Attitude determination of spacecraft usually utilizes vector measurements such as Sun, center of Earth, star, and magnetic field direction to update the quaternion which determines the spacecraft orientation with respect to some reference coordinates in the three dimensional space. These measurements are usually processed by an extended Kalman filter (EKF) which yields an estimate of the attitude quaternion.

Two EKF versions for quaternion estimation were presented in the literature; namely, the multiplicative EKF (MEKF) and the additive EKF (AEKF). In the multiplicative EKF it is assumed that the error between the correct quaternion and its a-priori estimate is, by itself, a quaternion that represents the rotation necessary to bring the attitude which corresponds to the a-priori estimate of the quaternion into coincidence with the correct attitude. The EKF basically estimates this quotient quaternion and then the updated quaternion estimate is obtained by the product of the a-priori quaternion estimate and the estimate of the difference quaternion. In the additive EKF it is assumed that the error between the a-priori quaternion estimate and the correct one is an algebraic difference between two four-tuple elements and thus the EKF is set to estimate this difference. The updated quaternion is then computed by adding the estimate of the difference to the a-priori quaternion estimate.

If the quaternion estimate converges to the correct quaternion, then, naturally, the quaternion estimate has unity norm. This fact was utilized in the past to obtain superior filter performance by applying normalization to the filter measurement update of the quaternion. It was observed for the AEKF that when the attitude changed very slowly between measurements, normalization merely resulted in a faster convergence; however, when the attitude changed considerably between measurements, without filter tuning or normalization, the quaternion estimate diverged. However, when the quaternion estimate was normalized, the estimate converged faster and to a lower error than with tuning only.

In last year's symposium we presented three new AEKF normalization techniques and we compared them to the brute force method presented in the literature. The present paper presents the issue of normalization of the MEKF and examines several MEKF normalization techniques.

I. INTRODUCTION

The normalization of the attitude quaternion in the AEKF was presented in past work [1,2]. Several techniques were developed and briefly tested. Those techniques included the following: brute force normalization of the quaternion (BF), considering the normalized quaternion a 'pseudo-measurement' and updating the quaternion in the usual manner (QPM), considering the magnitude of the norm a 'pseudo-measurement' and updating the quaternion in the usual manner (MPM), and finally developing the AEKF algorithm with a normalized attitude matrix, or the 'linearized orthogonalized matrix' normalization (LOM). Each method was shown to improve the attitude estimate and to speed convergence of the filter.

Several normalization techniques are also presented for the MEKF. We found that normalization in the MEKF is necessary to avoid divergence, even when the attitude does not change considerably between

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measurements. In the MEKF there are three points in the update cycle at which normalization can be performed. We present the methods for each, along with QPM and MPM methods, developed for the MEKF.

Each of the AEKF and MEKF methods are tested with data from a spacecraft in which the attitude does not change considerably between measurements. Fine Sun sensor, Earth sensor, magnetometer, and gyro data are used from each spacecraft. Finally, the results of the MEKF normalization methods are compared to those of the AEKF. Tests using data from a spacecraft undergoing high turning rates are currently being conducted but were not ready for publication in this paper.

In the next section we summarize the use of the AEKF and MEKF for attitude determination. In section III we explain the role of quaternion normalization in the AEKF and MEKF. In the following sections we present each of the normalization methods for both filters. Test results using simulated Earth Radiation Budget Satellite (ERBS) and Upper Atmospheric Research Satellite (UARS) data are given in Section VI and the conclusions follow in Section VII.

II. THE EKF ALGORITHM

The EKF algorithm is based on the following assumed models

$$\text{System model: } \dot{\underline{X}} = \underline{f}(\underline{X}(t), t) + \underline{w}(t) \quad (1)$$

$$\text{Measurement model: } \underline{z}_k = \underline{h}_k(\underline{X}(t_k)) + \underline{v}_k \quad (2)$$

where: $\underline{X}(t)$ = state vector
 $\underline{w}(t)$ = zero mean white process
 \underline{v}_k = zero mean white sequence

The measurement update and the propagation of the state estimate and of the error covariance are performed as

$$\dot{\underline{X}}_k(+) = \dot{\underline{X}}_k(-) + K_k [\underline{z}_k - \underline{h}_k(\dot{\underline{X}}_k(-))] \quad (3)$$

$$P_k(+) = [I - K_k H_k] P_k(-) [I - K_k H_k]^T + K_k R_k K_k^T \quad (4)$$

$$\dot{\underline{X}}(t) = \underline{f}(\underline{X}(t), t) \quad (5)$$

$$\dot{P}(t) = F(\underline{X}(t), t)P(t) + P(t)F^T(\underline{X}(t), t) + Q(t) \quad (6)$$

$$\text{where: } F(\underline{X}(t), t) = \frac{f(\underline{X}(t), t)}{\underline{X}(t)} \quad \left| \quad \underline{X}(t) = \dot{\underline{X}}(t) \right.$$

$$H(\underline{X}(-)) = \frac{h(\underline{X}(t))}{\underline{X}(t)} \quad \left| \quad \underline{X}(t) = \dot{\underline{X}}(t) \right.$$

P_k = estimation error covariance matrix
 R_k = covariance of the white sequence, \underline{v}_k
 Q_k = spectral density matrix of the white process, \underline{w}_k
 K_k = gain matrix

The state vector is given as

$$\underline{X}^T = \begin{bmatrix} \underline{g}^T & \underline{b}^T \end{bmatrix} \quad (7)$$

where: \underline{g} = four quaternion components
 \underline{b} = three gyro bias components

Note that equation (3) is the combination of the following

$$\hat{\underline{x}}_k(+) = \hat{\underline{x}}_k(-) + \hat{\underline{x}}_k(t_k) \quad (8)$$

$$\hat{\underline{x}}_k(t_k) = K_k \underline{y}_k \quad (9)$$

$$\underline{y}_k = \underline{z}_k - \underline{h}_k(\hat{\underline{x}}_k(-)) \quad (10)$$

where: \underline{y}_k = effective measurement or residual

\underline{z}_k = actual measurement

$\underline{h}_k(\hat{\underline{x}}_k(-))$ = the estimate of the actual measurement

The relationship between (3) and (8) has been presented in past work [3]. The first four components of $\hat{\underline{x}}_k$ are corrections to the \underline{g} estimate by the EKF, denoted as $d\underline{g}$. These are added to $\underline{g}_k(-)$, the best estimate of \underline{g} , to give $\underline{g}_k(+)$. The remaining elements in $\hat{\underline{x}}_k$ are the corrections to the gyro bias which are also then added to the best estimate of the gyro bias.

In the MEKF the quaternion elements of \underline{x} are treated differently. The definition of \underline{x} is given as

$$\underline{x}^T(t_k) = \begin{bmatrix} \underline{a}^T, \delta \underline{b}^T \end{bmatrix} \quad (11)$$

where: $\underline{a}^T = \begin{bmatrix} \phi, \theta, \mu \end{bmatrix}$ = three small angles based on the assumption that the error quaternion is composed of three small angles (vector) and 1 (scalar)

$\delta \underline{b}$ = corrections to the gyro bias

The correction to the quaternion, given as $d\underline{g}_k$, is then constructed according to

$$d\underline{g}_k^T = [\frac{1}{4}\phi \mid \frac{1}{4}\theta \mid \frac{1}{4}\mu \mid 1] \quad (12)$$

and the quaternion is updated as

$$\underline{g}_k(+) = \underline{g}_k(-) d\underline{g}_k^{-1} \quad (13)$$

Whereas the gyro bias is updated according to (8). The updated gyro bias components and $\underline{g}_k(+)$ are augmented into the state vector (7). For further discussion of the MEKF see [4].

The dynamics for both filters has been presented extensively in previous work and will not be included here. For reference see [1,2,3].

III. THE ROLE OF QUATERNION NORMALIZATION

The state measurement update equations are given in (8) for the AEKF and in (12) for the MEKF. Unless convergence has been attained, the updated quaternion $\underline{g}(+)$, is not necessarily normal, even if $\underline{g}(-)$ is. We know, however, that the quaternion which the algorithm is trying to estimate is necessarily normal. We can then enforce normalization on $\underline{g}_k(+)$ with the hope that the enforcement of this quality of the correct quaternion will direct the estimated quaternion in the right track and will enhance its convergence. Indeed, it was found in the past [2,5] that normalization is helpful. In particular, it was found that when the attitude varies slowly between measurements, normalization, although not necessary, resulted in a faster convergence; however, when the attitude changed rapidly between measurements, either filter tuning or normalization were necessary to avoid divergence. The use of normalization is superior to tuning because, first, tuning involves a tedious trial and error process, second, tuning is not a robust solution, and third, with quaternion normalization the final attitude estimate is closer to the correct quaternion.

IV. AEKF NORMALIZATION TECHNIQUES

Following is a summary of the AEKF normalization methods. The details are given in [1].

4.1 Brute Force Normalization (BF)

After \hat{x}_k has been computed in (8) the quaternion part of the state is normalized as

$$\hat{q}_k^* = \hat{q}_k(+)/|\hat{q}_k(+)| \quad (14)$$

and is augmented into $\hat{x}_k(+)$. This method was first presented in [5], where it was shown that the operation performed in (14) is equivalent (to first order) to

$$\hat{q}_k^*(+) = [\hat{q}_k(-) + d\hat{q}_k(+)] - \hat{q}_k(-)\hat{q}_k^T d\hat{q}_k(+) \quad (15)$$

The final term, $-\hat{q}_k(-)\hat{q}_k^T d\hat{q}_k(+)$, is a residual term, not found in (8) that must be compensated for in the filter computations. This term is retained after the normalization is performed and accounted for in the next stage of the filter operation. This mode of normalization does not affect the covariance computation of the EKF [5]. This computation constitutes an outside interference in the EKF algorithm and adds a certain complication to the algorithm.

4.2 Quaternion Pseudo-measurement (QPM)

In this algorithm the updated quaternion, \hat{q}_k , is used to form a pseudo-measurement as follows

$$y_{n,k} = \hat{q}_k(+)/|\hat{q}_k(+)| \quad (16)$$

The pseudo-measurement $y_{n,k}$ is, of course, a normalized quaternion. A measurement update is performed based on this measurement. The relationship between the measurement $y_{n,k}$ and the state vector is formulated as

$$y_{n,k} = H_{n,k}x_k + n_{n,k} \quad (17)$$

where: $H_{n,k} = \text{diag}[1,1,1,1|0..0]$
 $n_{n,k}$ = white measurement error

The covariance, $R_{n,k}$, of $n_{n,k}$ is set to be the diagonal matrix

$$R_{n,k} = \text{diag} [r^1, r^2, r^2, r^2] \quad (18)$$

where r is a small number. By adjusting the value of r we determine the degree of the imposed normalization on $\hat{q}_k(+)$. The QPM is performed after the state update, so the a priori state estimate is $\hat{x}_k(+)$. The output of this update is the full state vector, not just the estimate of x which is the difference between \hat{x}_k and its estimate $\hat{x}_k(+)$. The state update is performed as

$$\hat{x}_k^*(+) = \hat{x}_k(+) + K_{n,k}[y_{n,k} - H_{n,k}\hat{x}_k(+)] \quad (19)$$

where $K_{n,k}$ is computed using the updated covariance which corresponds to $\hat{x}_k(+)$ and $H_{n,k}$ and $R_{n,k}$ above. The covariance is then recomputed according to (4) and the new state and covariance are propagated as before.

It is important that r be well tuned. If r is too small the filter will attempt to replace the quaternion estimate by the normalized quaternion. However, a small r increases the variance of the quaternion estimation error, and a high credibility is assigned to the normalized quaternion even when it is not yet the correct quaternion. New measurements are not allowed to alter the quaternion estimate and the filter is stuck on a wrong estimate. This required tuning gives the algorithm a disadvantage. This disadvantage is overcome when the following normalization scheme is used.

4.3 Magnitude Pseudo-measurement (MPM)

In this scheme we use the square of the quaternion Euclidean norm, whose magnitude is assumed to be 1, as the measurement; that is

$$z_{n,k} = 1 + v_{n,k} \quad (20)$$

where $v_{n,k}$ is assumed to be a white measurement noise with variance r . This measurement quantity is a non-linear function of the quaternion components. The effective measurement, $y_{n,k}$, is computed as

$$y_{n,k} = z_{n,k} - [\hat{q}(+)_{1,k}^2 + \hat{q}(+)_{2,k}^2 + \hat{q}(+)_{3,k}^2 + \hat{q}(+)_{4,k}^2] \quad (21)$$

Following the derivations of [1] this is rewritten as

$$y_{n,k} = 1 - |\hat{g}_{j,k}|^2 \quad (22)$$

and $R_{n,k} = r$. This method does not have the tuning problems of the QPM. A small r does not imply that the measurement of \underline{g} is precise, it implies that the measurement of $|\underline{g}|^2$ is precise. So the estimate of \underline{g} does not stick to a wrong value, since the variance of \underline{g} doesn't approach the value of r .

4.4 Linearized Orthogonalized Matrix (LOM)

When the quaternion is of unit length the attitude matrix, $A(\underline{g})$, is orthonormal. It was proven in [6] that

$$A^*(\underline{g}) = \frac{1}{|\underline{g}|^2} A(\underline{g}) \quad (23)$$

is orthonormal and is the closest orthonormal matrix to $A(\underline{g})$. Using $A^*(\underline{g})$ in the development of the AEKF, rather than $A(\underline{g})$, practically enforces normalization.

V. MEKF NORMALIZATION TECHNIQUES

The normalization methods developed for the MEKF are presented here. In contrast to the AEKF algorithm, normalization is essential in the MEKF to avoid divergence. The first three methods, discussed in the ensuing, force normalization during the update of the quaternion. The final two methods are pseudo-measurement techniques similar to those presented for the AEKF.

5.1 Forced Normalization

After $\hat{g}_k(+)$ has been computed in (13), normalization is forced as

$$\hat{g}_k^* = \hat{g}_k(+)/|\hat{g}_k(+)| \quad (24)$$

No compensation is performed because no consequent divergence of the MEKF has been reported in the literature [7]. We refer to this method as 'normalized \underline{q} '.

The next method of forced normalization is to normalize $d\hat{g}$ from (12). This is performed as

$$|d\hat{g}| = (dq_{1,k}^2 + dq_{2,k}^2 + dq_{3,k}^2 + 1)^{1/2} \quad (25)$$

$$\begin{aligned} dg_{i,k}^* &= dq_{i,k}(+)/|d\hat{g}| \\ dg_{4,k} &= 1/|d\hat{g}| \end{aligned} \quad (26)$$

The normalized dg_k^* is then used in (13) to compute \hat{g}_k . This method is referred to as 'normalized $d\hat{q}$ '.

The final method forces normalization of the three small angles which form the attitude portion of the MEKF state, given in (11). Each of the angles is scaled to yield

$$\hat{\theta}^* = 2\theta/[\theta^2 + \theta^2 + \mu^2 + 4]^{1/2} \quad (27a)$$

$$\hat{\theta}^* = 2\theta/[\theta^2 + \theta^2 + \mu^2 + 4]^{1/2} \quad (27b)$$

$$\tau^* = 2\tau / [\hat{\theta}^2 + \hat{\theta}^2 + \hat{\mu}^2 + 4]^{1/2} \quad (27c)$$

The elements of dg are computed as

$$dq_{1,k}^* = \frac{1}{2}\hat{\theta}^* \quad (28a)$$

$$dq_{2,k}^* = \frac{1}{2}\hat{\theta}^* \quad (28b)$$

$$dq_{3,k}^* = \frac{1}{2}\hat{\mu}^* \quad (28c)$$

$$dq_{4,k}^* = 2 / [\hat{\theta}^2 + \hat{\theta}^2 + \hat{\mu}^2 + 4]^{1/2} \quad (28d)$$

Performing the scaling given in (27) results in the dg^* given in (28) being normal. The normalized dg^* is then used in (13) to compute $g_k(+)$. This method will be called 'normalized alpha', in reference to the vector, \underline{g} , of small angles in (11).

These methods constitute an outside interference in the MEKF algorithm. The covariance matrix is not affected. The complication of compensation is not added since divergence was not detected.

5.2 Quaternion Pseudo-measurement (QPM)

In this method we normalize the small angles of (11) and use them as the 'pseudo-measurement'. The relationship between dg and the angles is given in (12) and is repeated here.

$$\hat{dq}_1 = \hat{\theta}/2 \quad \hat{dq}_2 = \hat{\theta}/2 \quad \hat{dq}_3 = \hat{\mu}/2 \quad \hat{dq}_4 = 1 \quad (29)$$

Normalizing dg gives

$$dq_i^* = \frac{\hat{dq}_i}{(\hat{dq}_1^2 + \hat{dq}_2^2 + \hat{dq}_3^2 + 1)} \quad (30)$$

Use (30) in (29) to obtain

$$\frac{1}{2}\hat{\theta}^* = \frac{1}{2}\hat{\theta} / [(\hat{\theta}/2)^2 + (\hat{\theta}/2)^2 + (\hat{\mu}/2)^2 + 1]^{1/2} \quad (31a)$$

$$\frac{1}{2}\hat{\theta}^* = \frac{1}{2}\hat{\theta} / [(\hat{\theta}/2)^2 + (\hat{\theta}/2)^2 + (\hat{\mu}/2)^2 + 1]^{1/2} \quad (31b)$$

$$\frac{1}{2}\hat{\mu}^* = \frac{1}{2}\hat{\mu} / [(\hat{\theta}/2)^2 + (\hat{\theta}/2)^2 + (\hat{\mu}/2)^2 + 1]^{1/2} \quad (31c)$$

or

$$\hat{\theta}^* = p\hat{\theta}, \quad \hat{\theta}^* = p\hat{\theta}, \quad \hat{\mu}^* = p\hat{\mu} \quad (32)$$

where $p = 2(\hat{\theta}^2 + \hat{\theta}^2 + \hat{\mu}^2 + 4)^{-1/2}$

Note that dq_4 is not a part of the filter state. We assign it a value such that dg will be normal after the QPM update. Following is a summary of the algorithm computations in the order in which they are performed by the filter.

First p from (32) is computed using the updated angles of (10). The pseudo-measurement \underline{z} is then computed as

$$z_1 = p\hat{\theta} \quad z_2 = p\hat{\theta} \quad z_3 = p\hat{\mu} \quad (33)$$

The vector \underline{z} is related to the state vector as $\underline{z} = H_n \underline{x} + \underline{n}_n$, where \underline{x} is given in (10). The measurement matrix, H_n , and the noise covariance matrix, R_n , are, therefore, defined as

$$H_n = [I_{3 \times 3} \mid 0_{3 \times 3}] \quad (34)$$

$$R = [\text{diag } r^2]_{3 \times 3} \quad (35)$$

where r is a small number. A Kalman update is performed and the new covariance matrix is computed as follows

$$K_n = P(+)^T H_n^T [H_n P(+) H_n^T + R_n]^{-1} \quad (36)$$

$$\hat{\underline{x}}^*(+) = \hat{\underline{x}}(+) + K_n [\underline{z} - H_n \hat{\underline{x}}(+)] \quad (37)$$

$$P^*(+) = (I - K_n H_n) P(+)(I - K_n H_n)^T + K_n R_n K_n^T \quad (38)$$

where $P(+)$ = updated covariance matrix (before normalization)

$\underline{x}(+)$ = results of measurement update given in (9),

with $x_1 = \hat{\phi}$, $x_2 = \hat{\theta}$, $x_3 = \hat{\mu}$

The elements of $d\hat{\underline{q}}^*$ are computed as

$$dq_1^* = \hat{\phi}^*/2 \quad dq_2^* = \hat{\theta}^*/2 \quad dq_3^* = \hat{\mu}^*/2 \quad (39)$$

where the angles, $\hat{\phi}^*$, $\hat{\theta}^*$, and $\hat{\mu}^*$ are the first three components of $\hat{\underline{x}}^*(+)$. The fourth element of $d\hat{\underline{q}}^*$ is computed as

$$dq_4^* = 2(\hat{\phi}^2 + \hat{\theta}^2 + \hat{\mu}^2 + 4)^{-1/2} \quad (40)$$

Finally, the quaternion is computed using (13).

$$\hat{\underline{g}}_k^*(+) = \hat{\underline{g}}_k^*(+) dq_k^{*-1} \quad (41)$$

This method exhibits the same tuning problems as the AEKF QPM. Here, too, it is important that the $\hat{\underline{g}}$ be well tuned to avoid getting the quaternion estimate stuck on the wrong value. Again this presents somewhat of a disadvantage for this method.

5.3 Magnitude Pseudo-measurement

This method uses the magnitude of the normalized angles (10) as the measurement. Recall from (32)

$$p = 2(\hat{\phi}^2 + \hat{\theta}^2 + \hat{\mu}^2 + 4)^{-1/2} \quad (42)$$

We use p to normalize the angles

$$\hat{\phi}_n = p\hat{\phi}, \quad \hat{\theta}_n = p\hat{\theta}, \quad \hat{\mu}_n = p\hat{\mu} \quad (43)$$

Following (11), we rewrite (44) as

$$\hat{\underline{g}}_n = p\hat{\underline{g}} \quad (44)$$

The magnitude of $\hat{\underline{g}}_n$ is related to the estimate of the individual angles as follows

$$|\hat{\underline{g}}_n|^2 = p^2(\hat{\phi}^2 + \hat{\theta}^2 + \hat{\mu}^2) \quad (45)$$

The measurement z is defined as

$$z = |\hat{\underline{g}}_n|^2 + n \quad (46)$$

The effective measurement to be processed by the MEKF is then given as

$$y = z - |\hat{\underline{g}}|^2 \quad (47)$$

We need to express y as a linear combination of the difference between $\hat{\underline{g}}_n$ and $\hat{\underline{g}}$. Substituting (46) into (47) yields

$$y = |\hat{\underline{g}}_n|^2 + n - |\hat{\underline{g}}|^2 \quad (48)$$

Define $\delta \underline{g}$ as

$$\hat{\underline{g}}_n = \hat{\underline{g}} + \delta \underline{g} = \begin{bmatrix} \hat{\phi} + \delta \hat{\phi} \\ \hat{\theta} + \delta \hat{\theta} \\ \hat{\mu} + \delta \hat{\mu} \end{bmatrix} \quad (49)$$

Substituting (49) into (48) gives

$$y = (\hat{\phi} + \delta \hat{\phi})^2 + (\hat{\theta} + \delta \hat{\theta})^2 + (\hat{\mu} + \delta \hat{\mu})^2 - (\hat{\phi}^2 + \hat{\theta}^2 + \hat{\mu}^2) - n \quad (50)$$

Neglecting squares of $\delta \hat{\phi}$, $\delta \hat{\theta}$, $\delta \hat{\mu}$ yields

$$y = 2\hat{\phi}\delta\hat{\phi} + 2\hat{\theta}\delta\hat{\theta} + 2\hat{\mu}\delta\hat{\mu} + n \quad (51)$$

or

$$y = [2\hat{\phi}, 2\hat{\theta}, 2\hat{\mu}] \begin{bmatrix} \delta\hat{\phi} \\ \delta\hat{\theta} \\ \delta\hat{\mu} \end{bmatrix} \quad (52)$$

This defines the measurement matrix, H_n , as

$$H_n = [2\hat{\phi}, 2\hat{\theta}, 2\hat{\mu}, 0, 0, 0] \quad (53)$$

The MPM algorithm is then carried out as follows.

First p^2 is computed and used to obtain y .

$$y = (p^2 - 1) \quad (54)$$

Then H_n is computed and a small value is assigned to r , the uncertainty corresponding to n of (46). A Kalman update is performed and the covariance is updated.

$$K_n = P(+)^T H_n^T / [H_n P(+) H_n^T + r] \quad (55)$$

$$\hat{\underline{x}}^*(+) = \hat{\underline{x}}(+) + K_n [y - H_n \hat{\underline{x}}(+)] \quad (56)$$

$$P^*(+) = (I - K_n H_n) P(+) (I - K_n H_n)^T + K_n R_n K_n^T \quad (57)$$

where $P(+)$ = updated covariance matrix (before normalization)

$\hat{\underline{x}}(+)$ = results of measurement update given in (9),

with $x_1 = \phi$, $x_2 = \theta$, $x_3 = \mu$

The normalized $d\hat{\underline{q}}^*$ is then constructed.

$$dq_1^* = \frac{1}{2}\hat{\phi}^*, dq_2^* = \frac{1}{2}\hat{\theta}^*, dq_3^* = \frac{1}{2}\hat{\mu}^* \quad (58)$$

Again, since dq_4^* is not a part of the state we assign it a value such that $d\hat{\underline{q}}^*$ will be normal after the MPM update.

$$dq_4^* = 2(\hat{\phi}^{*2} + \hat{\theta}^{*2} + \hat{\mu}^{*2} + 4)^{-1/2} \quad (59)$$

The quaternion is then updated according to (13).

$$\hat{\underline{g}}_k^*(+) = \hat{\underline{g}}_k^* dq_4^{*-1} \quad (60)$$

This method is not subject to the tuning problems of the QPM for the same reasons as those given above for the AEKF MPM.

VI. RESULTS

The algorithms presented in this paper were tested using clean, nominal simulated data from the Earth Radiation Budget Satellite (ERBS) and noisy simulated data from the Upper Atmospheric Research Satellite (UARS). Two UARS datasets were created. One contains simulated data from a nominal 1 revolution per orbit (RPO) attitude and the other contains a 0.5 deg/sec simulated yaw maneuver. The ERBS data is taken with the spacecraft in its nominal 1 RPO attitude. The initial attitude error was 5 degrees and the value of r for the QPM and MPM algorithms was 10^{-5} for both the AEKF and the MEKF. We studied the behavior of each algorithm early, which we refer to as the transient period, and after convergence was achieved, which we refer to as steady state. Note also that each of the figures included starts with the first update, not with the initial attitude error of 5 degrees.

We first compared the AEKF normalization algorithms. All of the methods, including not normalizing at all, converged quickly. Figure 1 shows the first 5 seconds using ERBS data. The BF converges the quickest and the LOM the slowest. The QPM and MPM are similar to not normalizing. Figure 2 shows the transient period using the 1 RPO UARS data. All the methods converge quickly; the QPM has a slightly lower initial RSS attitude error. In the steady state, all the methods, including not normalizing at all, achieved similar, low RSS attitude errors. Figure 3 shows results from the UARS yaw maneuver. The LOM has the lowest error.

For the MEKF, normalization was found to be essential. Figure 4 shows the MEKF transient results from the UARS 1 RPO data. All the methods converge quickly. The results of not normalizing don't converge as low as the normalization results, and beyond the 10 seconds shown begin to diverge. In steady state, all the normalization methods achieved low RSS attitude errors. Figure 5 shows results from the UARS yaw maneuver. The three BF methods are slightly better than the QPM and MPM methods. Figure 6 shows steady state results, using ERBS data, for the three BF methods. The 'normalized dq' and 'normalized alpha' results are slightly better than the 'normalized q' results.

Finally, the two filters were compared. Figure 7 shows the BF method for the AEKF versus the 'normalized dq' method for the MEKF, in the transient period, using ERBS data. The AEKF converges a little faster than the MEKF. Figure 8 shows the steady state results from the UARS yaw maneuver, comparing the AEKF LOM and BF to the MEKF 'normalized q'. The MEKF 'normalized q' method has a lower RSS attitude error. The results of these comparisons of the two filters, in both the transient and steady state periods, were found to be true for the other methods as well.

VII. CONCLUSIONS

We found that all of the normalization methods presented work well and yield comparable results. In the AEKF, normalization is not essential since the data chosen for the test does not have a rapidly varying attitude. In the MEKF, normalization is necessary to avoid divergence of the attitude estimate. When the spacecraft experiences low angular rates, all of the methods for each of the filters have similar behavior. The choice of which algorithm to select as superior depends on the complexity of each algorithm. The pseudo-measurement techniques, for both the AEKF and MEKF, blend the normalization into the Kalman filter algorithm, but they don't represent an actual physical measurement, and are therefore somewhat obscure in their derivation. In addition, the QPM method requires the added burden of tuning. The AEKF BF algorithm is complicated by the need to compensate. The LOM method blends naturally into the filter development, using a normalized attitude to derive the filter update equations. The LOM is the slowest to converge but achieves the lowest RSS attitude error. In the MEKF, the brute force technique of normalizing the quaternion is the easiest to implement and is the most straight forward, but the other two brute force techniques have slightly better performance. All of the algorithms will be further tested with data from UARS undergoing a high turning rate. This may help to determine which of the algorithms, for each of the filters, has the best performance and may further substantiate the claim that under high rates normalization helps speed convergence and eliminate the need for tuning.

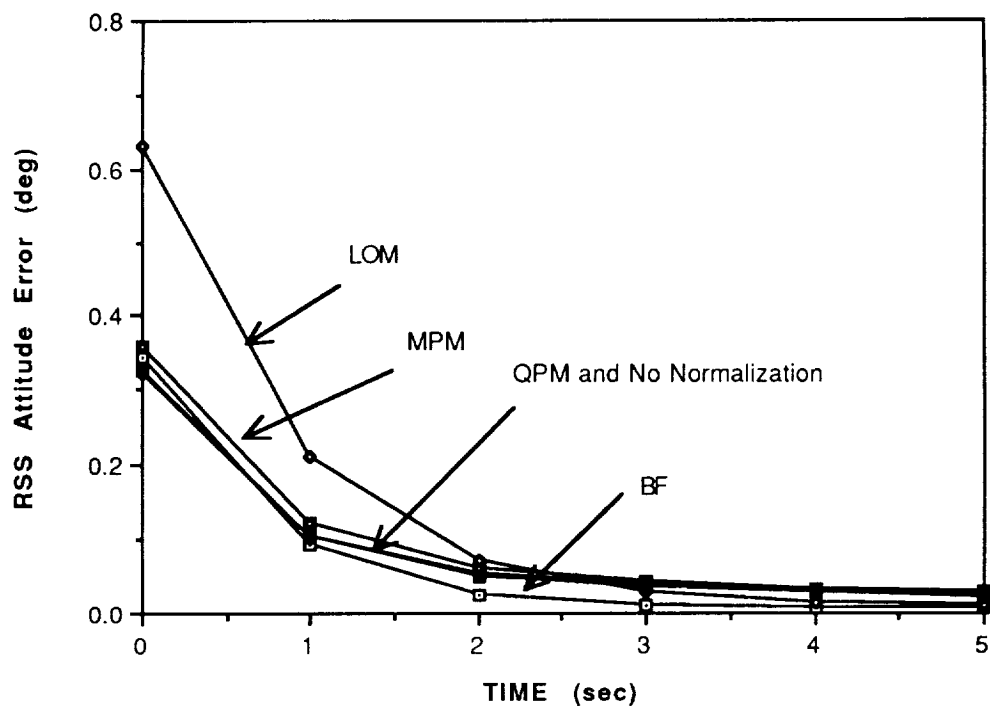


Fig. 1. ERBS AEKF: BF, QPM, MPM, LOM and No Normalization

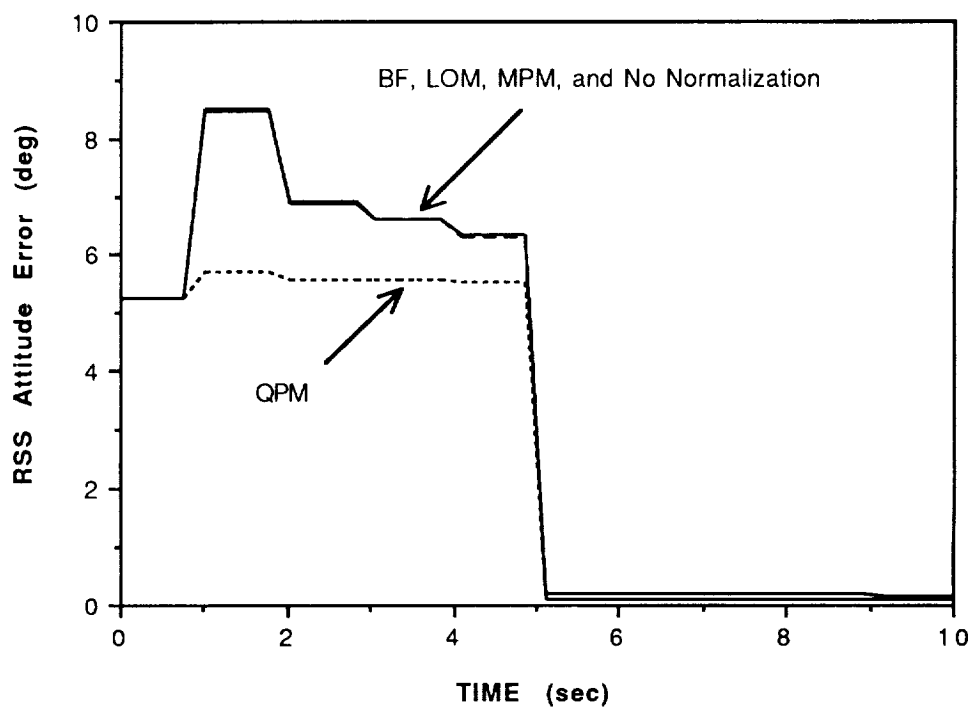


Fig. 2 UARS AEKF: Noisy Data, 1 RPO Attitude

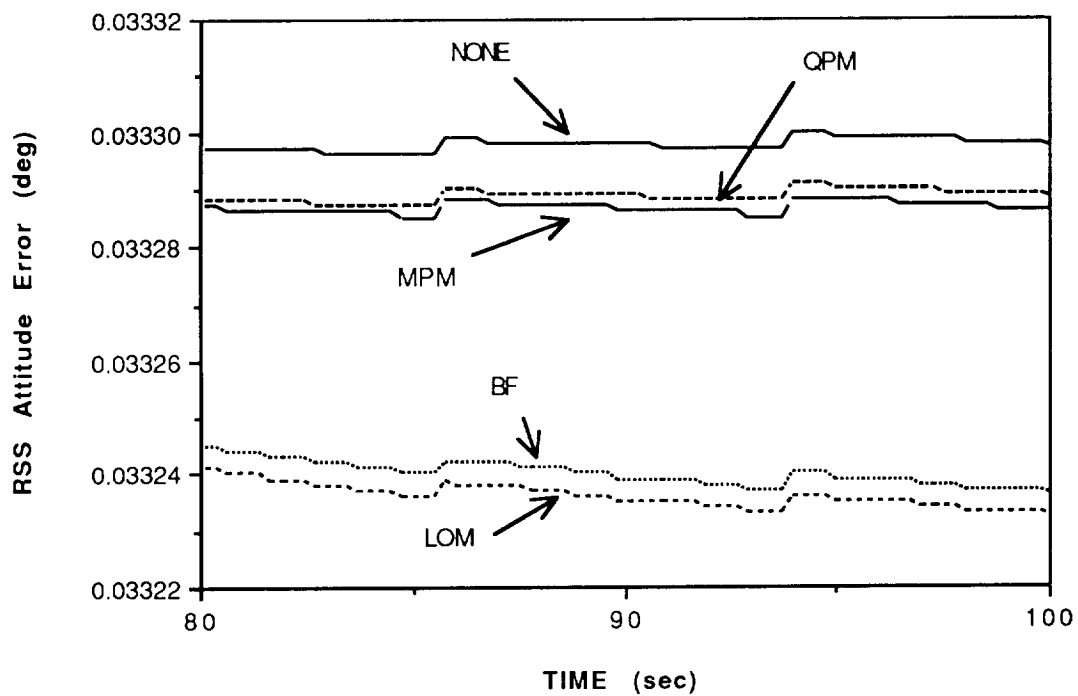


Fig. 3. UARS AEKF: Yaw Maneuver, Noisy Data

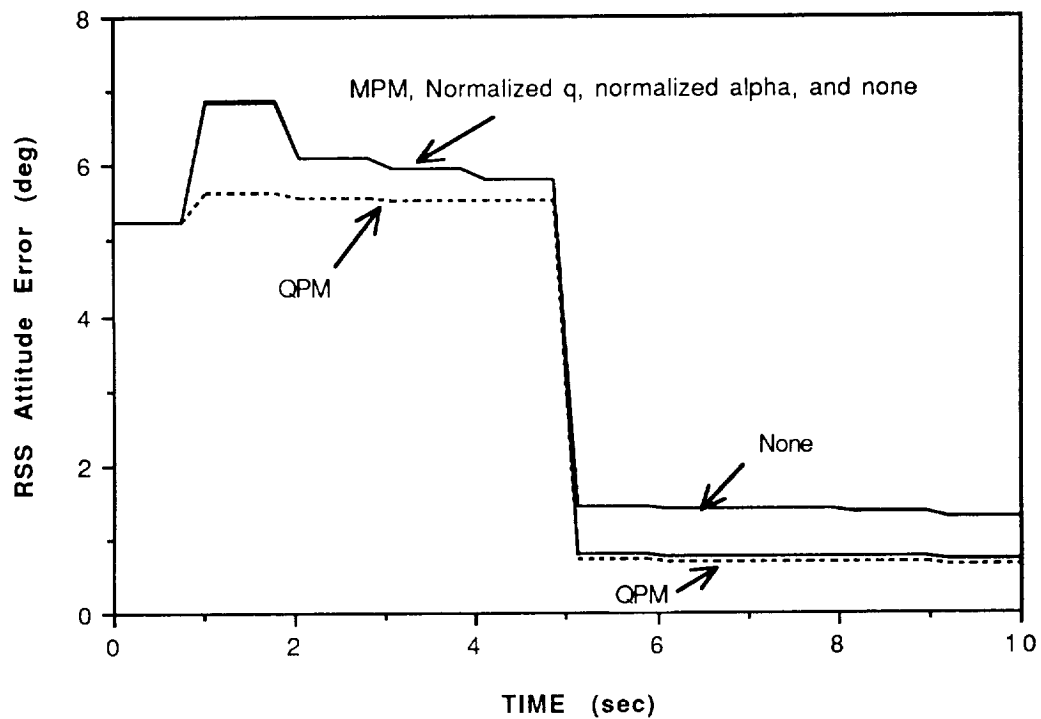


Fig. 4. UARS MEKF: Noisy Data, 1 RPO Attitude

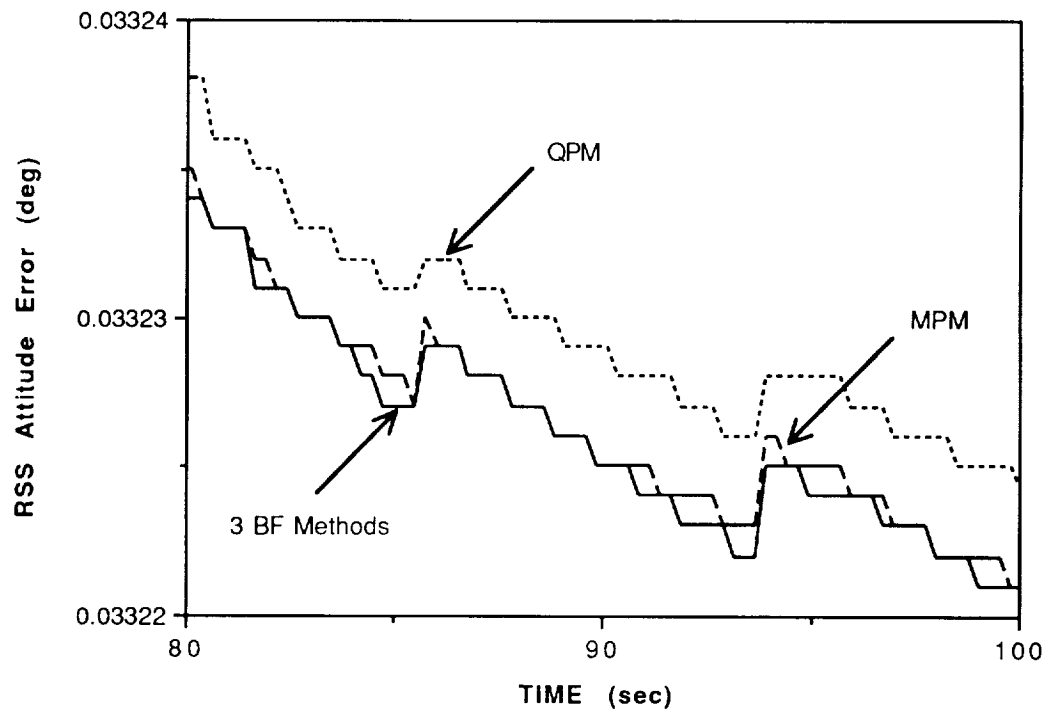


Fig. 5. UARS MEKF: Yaw Maneuver, Noisy Data

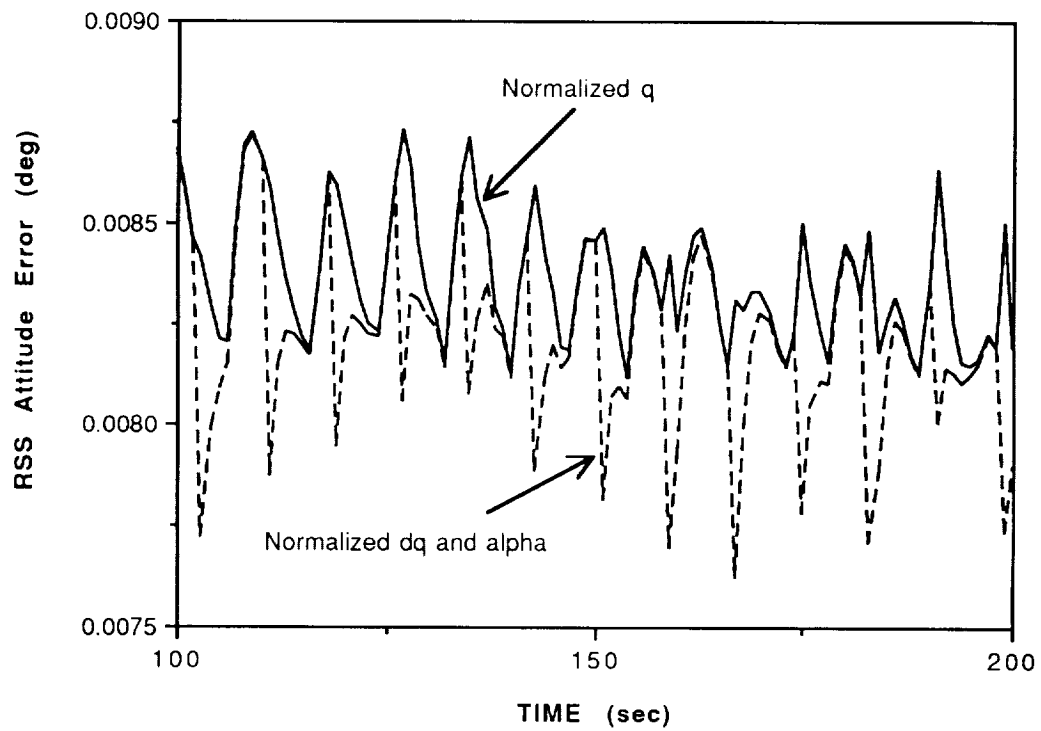


Fig. 6. ERBS MEKF: Normalized q, dq, and alpha

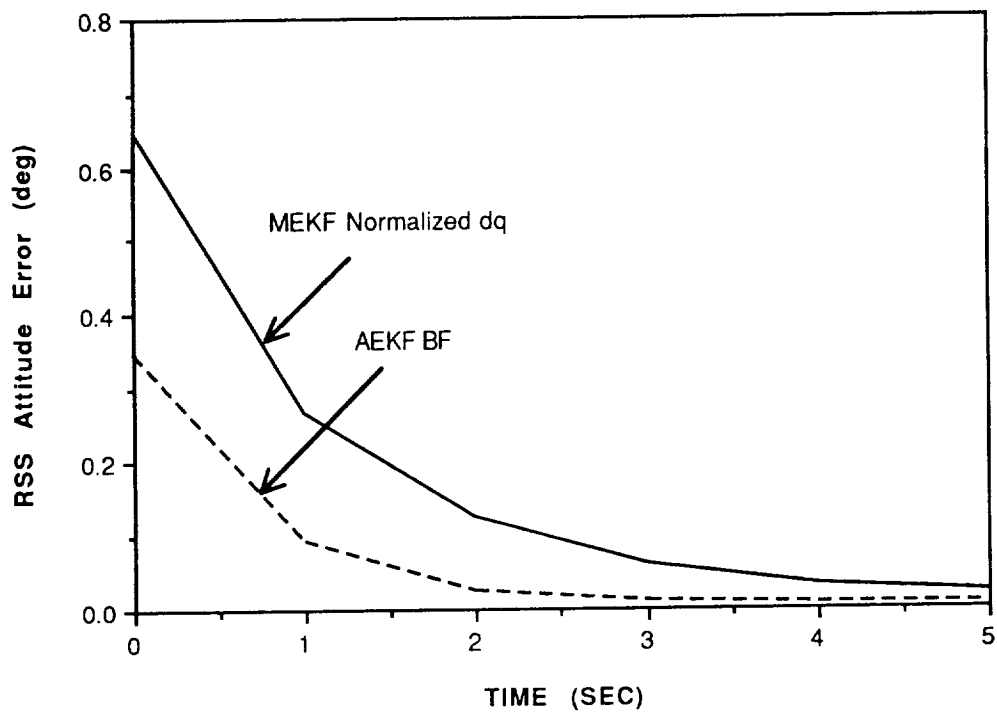


Fig. 7. ERBS BF Normalization: AEKF vs MEKF

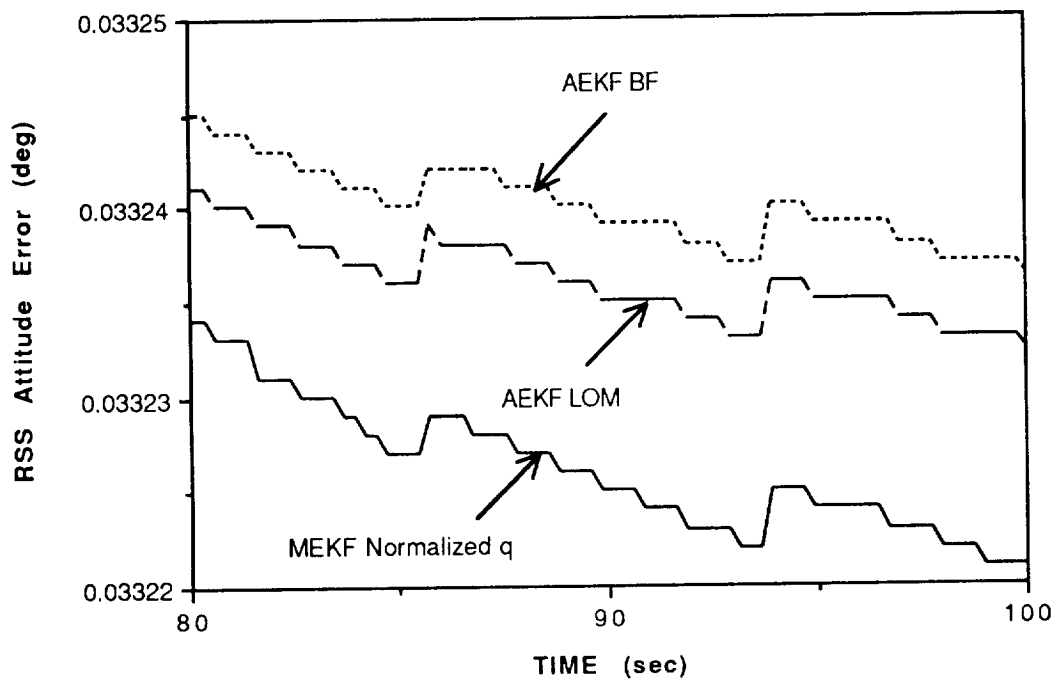


Fig. 8. UARS AEKF LOM,BF vs MEKF Normalized q: Yaw Maneuver, Noisy Data

IX. REFERENCES

1. Bar-Itzhack, I.Y. and Deutschmann, J., "Quaternion Normalization in Additive EKF for Spacecraft Attitude Determination", Flight Mechanics/Estimation Theory Symposium, NASA Goddard Space Flight Center, May 1991.
2. Bar-Itzhack, I.Y. and Deutschmann, J., "Extended Kalman Filter for Attitude Estimation of the Earth Radiation Budget Satellite", AAS Astrodynamics Conference, Portland, Oregon, Aug. 1990.
3. Deutschmann, J. and Bar-Itzhack, I.Y., "Extended Kalman Filter for Attitude Estimation of the Earth Radiation Budget Satellite", Flight Mechanics/Estimation Theory Symposium, NASA Goddard Space Flight Center, May 1989.
4. Bar-Itzhack, I.Y. and Harman, R.R., "True Covariance Simulation of the EUVE Update Filter", Flight Mechanics/Estimation Theory Symposium, NASA Goddard Space Flight Center, May 1989.
5. Bar-Itzhack, I.Y. and Oshman, Y., "Recursive Attitude Determination from Vector Observations: Quaternion Estimation", IEEE Transactions on Aerospace and Electronics Systems, Vol. AES-21, Jan. 1985, pp. 128-136.
6. Giardina, C.R., Bronson, R., and Wallen, I., "An Optimal Normalization Scheme", IEEE Transactions on Aerospace and Electronics Systems, Vol. AES-11, July 1975, pp. 443-446.
7. Murrell, J.W., "Precision Attitude Determination for Multimission Spacecraft", AIAA Guidance and Control Conference, Palo Alto, CA, August 1978.